Mathematical Challenges in Quantum Mechanics

February 8 – 13, 2016, Bressanone (Italy) Casa della Gioventù Universitaria

Programme & Abstracts



Monday 08/02/2016

- 08:30 09:00 Opening
- 09:00 10:00 Bernard Helffer On the time dependent Ginzburg-Landau system I
- 10:00 10:30 Coffee Break
- 10:30 12:30 Dimitri R. Yafaev Lectures on scattering theory I & II
- 12:30 14:30 Lunch Break
- 14:30 14:50 Costanza Benassi Decay of correlations in 2d quantum systems
- 14:50 15:10 Francesco C. De Vecchi Entropy Chaos and Bose-Einstein Condensation
- 15:10 15:20 Break
- 15:20 15:40 Giuseppe Genovese Gibbs Measures for the DNLS equation
- 15:40 16:00 Per Moosavi Non-equilibrium physics in a quenched Luttinger model
- 16:00 16:30 Coffee Break
- 16:30 16:50 Clément Tauber Topological edge states in two-gap unitary systems: a transfer matrix approach
- 16:50 17:10 Anna Vershynina Entanglement Rates in Bipartite Systems
- 17:10 17:30 Break
- 17:30 18:30 Sandro Stringari Gross-Pitaevskii equations for two-coupled Bose-Einstein condensates
- 18:30 Welcome Cocktail

Tuesday 09/02/2016

- 09:00 10:00 Dimitri R. Yafaev Lectures on scattering theory III
- 10:00 10:30 Coffee Break
- 10:30 12:30 Bernard Helffer On the time dependent Ginzburg-Landau system II & III
- 12:30 14:30 Lunch Break
- 14:30 14:50 Li Chen Multi-vortex Solutions of the Ginzburg-Landau Equations
- 14:50 15:10 Marko Erceg Characteristic length of sequences via one-scale H-measures
- 15:10 15:20 Break
- 15:20 15:40 Domenico Finco Schrödinger equation with nonlinear defect as effective model
- 15:40 16:00 Jean-Philippe Miqueu Spectral analysis of the magnetic Laplacian in the semiclassical limit
- 16:00 16:30 Coffee Break
- 16:30 16:50 Lorenzo Tentarelli NLS on graphs: recent results and perspectives
- 16:50 17:10 **Carlos Villegas-Blas** On the semiclassical limiting eigenvalue distribution for the hydrogen atom in a constant direction magnetic field
- 17:10 17:30 Lorenzo Zanelli Schrödinger dynamics and optimal transport of measures on the flat torus
- 17:30 17:50 Break
- 17:50 18:10 Max Lein Classification of topological insulators for classical light
- 18:10 18:30 **Davide Pastorello** Geometric description of finite-dimensional quantum systems in complex projective spaces and applications
- 18:30 18:50 Marcello Seri A sub-Riemannian Santaló formula and applications

Wednesday 10/02/2016

- 09:00 10:00 Dimitri R. Yafaev Lectures on scattering theory IV
- 10:00 10:30 Coffee Break
- 10:30 12:30 Jan Philip Solovej Bogoliubov theory for Bose and Fermi gases I & II
- 12:30 14:30 Lunch Break
- 14:30 14:50 Ivan Bardet Classical and Quantum parts of the dynamics in Open Quantum System
- 14:50 15:10 Leonid A. Borisov Equivalence by Chernoff for the methods of averaging of semigroups, generating by the Schrödinger-type operators
- 15:10 15:20 Break
- 15:20 15:40 Giuseppe De Nittis Topological nature of the Fu-Kane-Mele invariants
- 15:40 16:00 **Davide Fermi** Zeta regularization and the Casimir effect: a functional analytic framework
- 16:00 16:30 Coffee Break
- 16:30 17:30 Virginie Bonnaillie-Noël Magnetic Laplacian in sharp three-dimensional cones
- 17:30 17:50 Break
- 17:50 18:50 Federico Laudisa The issue of 'local realism' in the foundations of quantum mechanics

Thursday 11/02/2016

- 09:00 10:00 Jan Philip Solovej Bogoliubov theory for Bose and Fermi gases III
- 10:00 10:30 Coffee Break
- 10:30 12:30 Bernard Helffer On the time dependent Ginzburg-Landau system IV & V
- 12:30 14:30 Lunch Break
- 14:30 14:50 Niels Benedikter Mean-Field Dynamics of Fermionic Systems
- 14:50 15:10 Chiara Boccato Quantum many-body fluctuations around nonlinear Schrödinger dynamics
- 15:10 15:20 Break
- 15:20 15:40 Sébastien Breteaux The Time-Dependent Hartree-Fock-Bogoliubov Equations for Bosons
- 15:40 16:00 Marco Falconi Scattering theory in open quantum systems: Lindblad-type evolutions
- 16:00 16:30 Coffee Break
- 16:30 16:50 Markus Lange On asymptotic expansions for spin boson models
- 16:50 17:10 Alessandro Olgiati Effective dynamics for a multi-component mixture of Bose-Einstein condensates
- 17:10 17:30 Chiara Saffirio From the Hartree-Fock dynamics to the Vlasov equation
- 17:30 17:50 Break
- 17:50 18:50 Luis Vega An isoperimetric-type inequality for Dirac Hamiltonians with electrostatic shell interactions

Friday 12/02/2016

- 09:00 10:00 Bernard Helffer On the time dependent Ginzburg-Landau system VI
- 10:00 10:30 Coffee Break
- 10:30 12:30 Jan Philip Solovej Bogoliubov theory for Bose and Fermi gases IV & V
- 12:30 14:30 Lunch Break
- 14:30 14:50 Jean-Claude Cuenin Eigenvalue bounds for Dirac and fractional Schrödinger operators with complex potentials
- 14:50 15:10 Francesco Ferrulli Bounds for complex eigenvalues of single and bilayer graphene's Hamiltonian operator
- 15:10 15:20 Break
- 15:20 15:40 Orif Ibrogimov Essential spectrum of singular matrix differential operators
- 15:40 16:00 Markus Holzmann Approximation of Schrödinger operators with singular interactions supported on hypersurfaces
- 16:00 16:30 Coffee Break
- 16:30 16:50 Anton A. Kutsenko Periodic operators with defects of smaller dimension
- 16:50 17:10 **Thomas Ourmières-Bonafos** On the bound states of Schrödinger operators with δ -interactions on conical surfaces
- 17:10 17:30 Bartosch Ruszkowski Hardy inequalities on the Heisenberg group for convex bounded polytopes
- 17:30 17:50 Break
- 17:50 18:50 Pavel Exner Quantum systems changing abruptly their spectral properties

Saturday 13/02/2016

- 08:30 09:30 Jan Philip Solovej Bogoliubov theory for Bose and Fermi gases VI
- 09:30 10:30 Dimitri R. Yafaev Lectures on scattering theory V
- 10:30 11:00 Coffee Break
- 11:00 12:00 Dimitri R. Yafaev Lectures on scattering theory VI
- 12:00 13:00 **Paolo Antonelli** Finite energy analysis for some hydrodynamic systems arising in the description of quantum fluids

Titles and Abstracts

Courses

On the time dependent Ginzburg-Landau system

Bernard Helffer LMJL Université de Nantes

In this course we would like to discuss spectral properties of some non self-adjoint operators appearing in the analysis of the long time behavior of the solutions of the time dependent Ginzburg Landau system (due to Eliashberg-Gorkov) and then to consider the question of the global stability of the stationary normal solutions in presence of an electric current flowing through a two-dimensional wire.

We show that when the current is sufficiently strong the solution converges in the long-time limit to the normal state. We provide two types of upper bounds for the critical current where such global stability is achieved: by using the principal eigenvalue of the magnetic Laplacian associated with the normal magnetic field, and through the norm of the resolvent of the linearized steady-state operator. In the latter case we estimate the resolvent norm in large domains by the norms of approximate operators defined on the plane and the half-plane. We also obtain a lower bound, in large domains, for the above critical current by obtaining the current for which the normal state looses its local stability.

The recent results presented in the course were obtained in collaboration with Y. Almog or X. Pan. Introductory books for the time independent problems could be the monographs of Sandier-Serfaty and Fournais-Helffer (both in the series Progress in non linear analysis – Birkhäuser).

- 1. Y. Almog. *The stability of the normal state of superconductors in the presence of electric currents*. SIAM Journal on Mathematical Analysis, 40 (2008), pp. 824–850.
- 2. Y. Almog and B. Helffer. *Global stability of the normal state of superconductors in the presence of a strong electric current*. Comm. in Math. Physics 2014.
- 3. Y. Almog and B. Helffer. *On the spectrum of non-selfadjoint Schrödinger operators with compact resolvent.* arXiv:1410.5122. Comm. in PDE 2015.
- 4. Y. Almog, B. Helffer and X. B. Pan. *Superconductivity near the normal state under the action of electric currents and induced magnetic fields in* ℝ², Commun. Math. Phys. 300 (2010), 147–184.
- 5. Y. Almog, B. Helffer and X. B. Pan. *Superconductivity near the normal state in a half-plane under the action of a perpendicular electric current and an induced magnetic field*, Trans. AMS 365 (2013), 1183–1217.

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- 7. Y. Almog, B. Helffer and X. B. Pan. *Mixed normal-superconducting states in the presence of strong electric currents.* ArXiv:1505.06327 (2015).
- 8. P. Bauman, H. Jadallah and D. Phillips. *Classical solutions to the time-dependent Ginzburg-Landau equations for a bounded superconducting body in a vacuum*, J. Math. Phys., 46 (2005), 095104.
- 9. Z. M. Chen, K.-H. Hoffmann, and J. Liang. *On a nonstationary Ginzburg-Landau superconductivity model*, Math. Methods Appl. Sci., 16 (1993), pp. 855–875.
- 10. R. Dautray and J.L. Lions. *Mathematical Analysis and Numerical Methods for Science and Technology*. Vol. 5, Evolution problems I, Springer- Verlag, 1988.
- 11. Q. Du. Global existence and uniqueness of solutions of the time-dependent Ginzburg-Landau model for superconductivity, Appl. Anal., 53 (1994), pp. 1–17.
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- 13. K.J. Engel and R. Nagel. *One-parameter semigroups for linear evolution equations*. Graduate texts in Mathematics 194, Springer.
- 14. L. C. Evans. Partial Differential Equations, AMS, 1st. ed., 1998.
- 15. E. Feireisl and P. Takáč. Long-time stabilization of solutions to the Ginzburg-Landau equations of superconductivity, Monatsh. Math., 133 (2001), pp. 197– 221.
- 16. J. Fleckinger-Pellé, H. G. Kaper, and P. Takáč. *Dynamics of the Ginzburg-Landau equations of superconductivity*, Nonlinear Anal., 32 (1998), pp. 647–665.
- 17. S. Fournais and B. Helffer. *Spectral Methods in Surface Superconductivity*, Birkhäuser, 2009.
- 18. T. Giorgi and D. Philips. *The breakdown of superconductivity due to strong fields for the Ginzburg-Landau model*, SIAM J. Math. Anal., 30 (1999), pp. 341–359.
- 19. P. Grisvard. *Elliptic problems in nonsmooth domains*, vol. 24 of Monographs and Studies in Mathematics, Pitman (Advanced Publishing Program), Boston, MA, 1985.
- 20. P. Grisvard. *Singularities in boundary value problems*. Springer (1992).
- 21. B. Helffer, On pseudo-spectral problems related to a time dependent model in superconductivity with electric current. Confluentes Math. 3, No. 2, 237–251 (2011).

- 22. B. Helffer. *Spectral theory and its applications*. Cambridge University Press 2013.
- 23. B. Helffer and J. Sjöstrand. *From resolvent bounds to semigroup bounds*. Preprint : arXiv:1001.4171v1, (2010).
- 24. R. Henry. Master's memoir (2010).
- R. Henry. Spectral instability of some non-selfadjoint anharmonic oscillators. C. R. Acad. Sci. Paris, Ser. I 350 (2012) 1043–1046.
- 26. R. Henry. Spectral instability for the complex Airy operator and even non-selfadjoint anharmonic oscillators. To appear in Journal of Spectral Theory.
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- 28. B. I. Ivlev and N. B. Kopnin. *Electric currents and resistive states in thin superconductors*, Advances in Physics, 33 (1984), pp. 47–114.
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Bogoliubov theory for Bose and Fermi gases

Jan Philip Solovej University of Copenhagen

The lectures will start with quasi-free states for fermions and bosons, quadratic Hamiltonians, Bogoliubov diagonalization again for fermions and for the more difficult case of bosons, and entropy of quasi-free states. This will lead to the definition of the Bogoliubov variational model, which is arrived at by restricting to quasi-free states. The particular case of translation invariant models will be discussed. This will lead to the BCS model for Fermions and the Bogoliubov model for Bosons. The lecture will discuss the Bose case in more details.

Lectures on scattering theory

Dimitri R. Yafaev IRMAR, Université de Rennes I

The goal is to introduce fundamental notions of mathematical scattering theory and to describe its quantum mechanical applications (long-range and multiparticle scattering).

A part of these lectures can be found on ArXiv as well as on https://maths-proceedings.anu.edu.au/040/CMAproc40-yafaev.pdf

Plenary Talks

Finite energy analysis for some hydrodynamic systems arising in the description of quantum fluids

Paolo Antonelli Gran Sasso Science Institute

Quantum Hydrodynamic systems arise in the description of superfluidity, Bose-Einstein condensation or in the modeling of semiconductor devices. They are described by a compressible, dispersive, inviscid flow. In this talk I will review some recent results on the global existence of finite energy weak solutions to the QHD system. The strategy of our proof is based on a polar factorisation technique which allows us to deal with the mass and current densities without the need of defining the velocity field in the nodal regions. I will then discuss some hydrodynamic models for superfluidity at finite temperatures, related to the Landau two-fluid formulation, where the superfluid, quantum part interacts with the normal viscous part of the fluid. I will present some partial results and try to explain the difficulties arising in the general model.

Those are results obtained jointly with P. Marcati.

Magnetic Laplacian in sharp three-dimensional cones

Virginie Bonnaillie-Noël ENS Paris

In this talk, we are interested in the bottom of the spectrum of Magnetic Laplacian in sharp three-dimensional cones.

In the case of circular cones, we prove a complete asymptotic expansion as the angle tends to zero and highlight the influence of the magnetic field.

For general cone, we prove an upper bound for the ground state energy of the magnetic Laplacian with constant magnetic field on cones that are contained in a half-space. This bound involves a weighted norm of the magnetic field related to moments on a plane section of the cone. When the cone is sharp, i.e. when its section is small, this upper bound tends to 0. A lower bound on the essential spectrum is proved for families of sharp cones, implying that if the section is small enough the ground state energy is an eigenvalue. This circumstance produces corner concentration in the semi-classical limit for the magnetic Schrödinger operator when such sharp cones are involved.

This is a joint work with M. Dauge, N. Popoff and N. Raymond.

Quantum systems changing abruptly their spectral properties

Pavel Exner

Doppler Institute for Mathematical Physics and Applied Mathematics, Prague

The aim of this talk is to analyze several classes of Schrödinger operators with potentials that are below unbounded but their negative part is localized in narrow channels. They have the common property that they exhibit an abrupt parameter-dependent spectral transition: if the coupling constant exceeds a critical value the spectrum will cover the whole real axis, corresponding to the particle escape to infinity. A prototype of such a behavior can be found in the so-called Smilansky model devised to illustrate that an irreversible behavior is possible even if the heat bath to which the systems is coupled has a finite number of degrees of freedom. We review its properties and analyze a regular version of this model, as well as another system in which $x^p y^p$ potential is amended by a negative radially symmetric term; in the latter case the subcritical spectrum is purely discrete.

The results come from a common work with Diana Barseghyan, Vladimir Lotoreichik and Miloš Tater.

The issue of 'local realism' in the foundations of quantum mechanics

Federico Laudisa Università degli Studi di Milano Bicocca

According to a widespread view, the Bell theorem establishes the untenability of so-called 'local realism' for quantum mechanics. In my talk I will briefly review the issue of local realism, starting from the Einsteinian critical stance toward quantum mechanics up to the Bell achievements and I will claim that the above view is incorrect: the Bell theorem does not concern realism but simply locality, and realism is rather a general philosophical framework that per se cannot be refuted in itself by any quantum experiment. As a consequence, realism in quantum mechanics is not something that can be simply explained away once and for all on the basis of experiments, but rather something that must be conceptually characterized and discussed in terms of its foundational virtues and vices.

Gross-Pitaevskii equations for two-coupled Bose-Einstein condensates

Sandro Stringari Università di Trento

In this talk I will summarize some recent theoretical advances concerning the solutions of the Gross-Pitaevskii equation for two-component Bose-Einstein condensates. Special emphasis will be given, in particular, to the nonlinear solutions of solitonic nature, both in the presence and in the absence of coherent coupling between the two condensates. I will also outline possible experimental perspectives in the field.

An isoperimetric-type inequality for Dirac Hamiltonians with electrostatic shell interactions

Luis Vega UPV and BCAM

I shall present some recent work in collaboration with N. Arrizabalaga and A. Mas about the spectral properties of the coupling H + aV, where H is the massive free Dirac operator in 3d, and aV is an electrostatic shell potential (which depends on the coupling real parameter a) located on the boundary of a smooth domain. Our main result is an isoperimetric-type inequality for the admissible range of a's for which the coupling H + aV generates pure point spectrum in (m, m). That the ball is the unique optimizer of this inequality is also shown. Regarding some ingredients of the proof, we make use of the Birman-Schwinger principle adapted to our setting in order to prove first some monotonicity property of the admissible a's, and then a sharp constant of a quadratic form inequality, from which the isoperimetric-type inequality is derived.

Contributed Talks

Decay of correlations in 2d quantum systems

Costanza Benassi University of Warwick

The study of the decay of the relevant two point correlation functions of a lattice system provides us with a great deal of information about the presence of spontaneous symmetry breaking and phase transitions. As theorised firstly by Mermin and Wagner (1966), no continuous symmetry breaking should be found in d \leq 2. In this work, we generalise the results of McBryan-Spencer (1977) and Koma-Tasaki (1992) and we prove power-law decay of correlation functions in a very general setting, provided that the Hamiltonian has a U(1) symmetry. Good examples are the quantum XXZ model, Hubbard and t-J models and random loop models.

This talk is based on a joint work with J. Fröhlich and D. Ueltschi.

Mean-Field Dynamics of Fermionic Systems

Niels Benedikter University of Copenhagen

Making predictions from the time-dependent Schrödinger equation for many interacting particles is very difficult. Therefore one is interested in deriving simpler effective evolution equations approximating the Schrödinger equation.

We consider the mean-field regime of fermionic systems and derive time-dependent Hartree-Fock theory. We point out a semiclassical structure which is crucial for controlling the approximation for long times. Our method is based on fermionic Bogoliubov transformations.

The result can be extended to mixed states, e.g. systems at positive temperature. The joint semiclassical limit yields the Vlasov equation as a second step of approximation.

Joint work with Vojkan Jakšić, Marcello Porta, Chiara Saffirio, and Benjamin Schlein.

- 1. N. Benedikter, M. Porta, and B. Schlein. Mean-Field Evolution of Fermionic Systems. *Comm. Math. Phys.*, 331:1087–1131, 2014.
- 2. N. Benedikter, M. Porta, and B. Schlein. Mean-Field Dynamics of Fermions with Relativistic Dispersion. *J. Math. Phys.*, 55(2):021901, 2014.
- 3. N. Benedikter, V. Jakšić, M. Porta, C. Saffirio, and B. Schlein. Mean-field Evolution of Fermionic Mixed States. to appear in *Comm. Pure Appl. Math.*
- 4. N. Benedikter, M. Porta, C. Saffirio, and B. Schlein. From the Hartree dynamics to the Vlasov equation. *ArXiv* 1502.04230, 2015.

Quantum many-body fluctuations around nonlinear Schrödinger dynamics

Chiara Boccato University of Zurich

We consider a system of bosons interacting through a two-body potential $N^{3\beta-1}V(N^{\beta}x)$, scaling with the number of particles N. When a very large number of particles is involved, it is difficult to calculate physical quantities from the time-dependent Schrödinger equation. However, it is possible to approximate the dynamics by a one-particle non-linear Schrödinger equation; the reliability of this approximation is the problem we want to investigate.

Our work focuses on fluctuations around the nonlinear Schrödinger dynamics. In a second-quantized setting, for $0 < \beta < 1$, we construct a unitary evolution with quadratic generator, which, in the limit of large N, allows us to approximate the fully evolved state in Fock space norm.

Joint work with Serena Cenatiempo and Benjamin Schlein.

1. C. Boccato, S. Cenatiempo and B. Schlein. Quantum many-body fluctuations around nonlinear Schrödinger dynamics. Preprint arXiv:1509.03837.

Equivalence by Chernoff for the methods of averaging of semigroups, generating by the Schrödinger-type operators

Leonid A. Borisov Keldysh Institute of Applied Mathematics RAS

The method of study one-parameter semigroups (generated by the evolutionary differential equations), and statistical equilibrium density operators (in classical and quantum Statistical Mechanics problems) based on Feynmann formula [1] and the path integration was developed by several authors. The papers [2,3] are essental in the development of the mathematical basis for this method. Development of this method in recent times is associated with obtaining the explicit analytical expressions [4] for the solving operators of the Cauchy problem for differential equations such as the Heat equation or Schrödinger equation. The fact underlying these studies is the Chernoff theorem [5]. This theorem establishes the convergence of the certain iterative process to the semigroup, which is the solution of the Cauchy problem. My goal is to obtain representations for fractional diffusion operator kernels, the operator of the relativistic Hamiltonian and kernels corresponding equilibrium density operators and operators of quantum dynamics.

To construct the semigroups which will be the solutions of evolutionary differential equations such as the diffusion equation, fractional order diffusion equation, Schrödinger equation with relativistic Hamiltonian, I make use of the procedure of averaging of the operator-valued functions.

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- 2. O.G. Smolyanov, A.G. Tokarev, A. Truman. *Hamiltonian Feynman path integrals via the Chernoff formula*, J. Math. Phys. 43:10 (2002), 5161–5171.
- 3. E. Nelson, *Feynman integrals and the Schrodinger equation*, J. Math. Phys. 5:3 (1964), 332–343.
- 4. L.A. Borisov, Yu.N. Orlov. Analyzing the dependence of finite-fold approximations of the harmonic oscillator equilibrium density matrix and of the Wigner function on the quantization prescription. TMPh 184:1 (2015), 986–995,
- 5. P. Chernoff. *Note on product formulas for operator semigroups*, J. Funct. Anal. 84 (1968), 238–242.

The Time-Dependent Hartree-Fock-Bogoliubov Equations for Bosons

Sébastien Breteaux BCAM - Basque Center for Applied Mathematics

We use quasifree reduction to derive the time-dependent Hartree-Fock-Bogoliubov (HFB) equations describing the dynamics of quantum fluctuations around a Bose-Einstein condensate in \mathbb{R}^d . We prove global well-posedness for the HFB equations for sufficiently regular pair interaction potentials, and establish key conservation laws. Moreover, we show that the solutions to the HFB equations exhibit a symplectic structure, and have a form reminiscent of a Hamiltonian system. In particular, this is used to relate the HFB equations to the HFB eigenvalue equations encountered in the physics literature.

Joint work with V. Bach, T. Chen, J. Fröhlich, and I.-M. Sigal.

Multi-vortex Solutions of the Ginzburg-Landau Equations

Li Chen University of Toronto - ETH Zürich

I will introduce the Ginzburg-Landau (GL) equations and give a very brief discussion of solutions with a single vortex per lattice cell. The focus of this talk, however, will be on the general case of multi-vortex solutions. We attempt to bifurcate a branch of such solutions from the normal state solution with constant magnetic field. A main difficulty is the reduction of dimension of solutions of the linearized problem to the lowest possible value so that one can more easily understand the location of vortices of these solutions. One can transfer this problem onto a suitable space of theta functions and use more algebraic methods to study the problem. I will discuss low flux (per lattice cell) results and give a brief sketch of the proof by exploiting symmetries of the underlying Abrikosov lattice.

Eigenvalue bounds for Dirac and fractional Schrödinger operators with complex potentials

Jean-Claude Cuenin LMU München

We present new Lieb-Thirring type bounds for fractional Schrödinger operators and Dirac operators with complex-valued potentials. The main new ingredient is a resolvent bound in Schatten spaces for the unperturbed operator.

Topological nature of the Fu-Kane-Mele invariants

Giuseppe De Nittis Pontificia Universidad Católica de Chile

Quantum systems protected by an odd time-reversal (pseudo-)symmetry (TRS) play a crucial role in the theory of topological quantum systems (e.g. topological insulators, photonic crystals, adiabatically perturbed quantum systems, etc.). In the case of ordinary condensed matter electronic systems with an odd TRS (class All topological insulators) the topologically protected phases are described by an invariant known as Fu-Kane-Mele index. The construction of this invariant, in its original form, turns out to be specific for the particular model and not immediately generalizable. On the other hand the description of odd TRS invariant systems (in absence of disorder) can be understood inside the classification theory of "Quaternionic" vector bundles. In this talk I will describe an intrinsic topological invariant for "Quaternionic" vector bundles, called FKMM-invariant, which generalizes and explains the topological nature of the Fu-Kane-Mele index. I will show that the FKMM-invariant is a universal characteristic class which can be defined for "Quaternionic" vector bundles in full generality and has the virtue to be a complete invariant for low-dimensional systems endowed with any type of odd TRS. As a special application I will describe how the FKMM-invariant completely classifies the topological phases over all possible types of low-dimensional involutive spheres and tori and I will compare our classification with recent results in the literature concerning two-dimensional adiabatically perturbed quantum systems.

Joint work with K. Gomi.

Entropy Chaos and Bose-Einstein Condensation

Francesco C. De Vecchi Università degli Studi di Milano

We prove the entropy-chaos property for the system of N undistinguishable interacting diffusions rigorously associated, through the Nelson map, with the ground state of N trapped Bose particles in the Gross-Pitaevskii scaling limit of infinite particles. On the path-space we show that the sequence of probability measures of the one-particle interacting diffusion weakly converges to a limit probability measure, uniquely associated with the minimizer of the Gross-Pitaevskii functional. The talk is based on joint work with Sergio Albeverio and Stefania Ugolini (see arXiv:1512.04729).

Characteristic length of sequences via one-scale H-measures

Marko Erceg University of Zagreb

By Vitali's theorem, the lack of strong convergence of L^2 bounded sequences is caused by *oscillations* and/or *concentration* effects, which motivates the study of these effects. Although the direction of oscillations and the point of concentrations can be detected using H-measures (also called microlocal defect measures), sometimes this is not satisfactory since we are also interested in the scale of this phenomena (e.q. frequency of oscillations). We introduce (ω_n) -concentrating property which, together with the known (ω_n) -oscillating property, can give a full picture of the scale of observed sequences. These conditions also illustrate when H-measures and semiclassical measures (also called Wigner measures) coincide. Furthermore, we would like to present a more recent variant of microlocal defect functionals, *one-scale H-measures*, which were introduced as a generalization of H-measures with a characteristic length, being also an extension of semiclassical measures. Using the framework of one-scale H-measures, we develop a variant of compensated compactness suitable for partial differential equations with a characteristic length.

This is a joint work with NENAD ANTONIC and MARTIN LAZAR.

Scattering theory in open quantum systems: Lindblad-type evolutions

Marco Falconi

Institut für Analysis, Dynamik und Modellierung, Universität Stuttgart

In this talk I will discuss the long time asymptotics for open quantum systems of Lindblad-type. The exemplifying model is the quantum dynamics of a particle scattered off a dynamical target, the latter occupying a compact region of space. Under suitable assumptions on the interaction, we are able to prove the existence of wave operators, and asymptotic completeness (in a suitable sense). Even if the Lindblad operator acts on the trace class ideal, one of the main ingredients for the proof is a theory of scattering for dissipative operators in Hilbert spaces.

Based on a joint work with J. Faupin, J. Fröhlich, and B. Schubnel.

Zeta regularization and the Casimir effect: a functional analytic framework

Davide Fermi Università degli Studi di Milano

In quantum field theory there often appear ill-defined expressions; these can be assigned a meaning in terms of analytic continuation using the Zeta approach. Having in mind these applications to QFT, I will first introduce an abstract framework where the powers of some given, positive self-adjoint operator are used to define an infinite scale of Hilbert spaces; this framework allows, in particular, to address with a natural language the topic of integral kernels (focusing, especially, on their regularity and on their analytic continuation with respect to some parameters). Next, using this language, I will consider the canonical quantization of a scalar field living on an arbitrary spatial domain, with a classical background potential. In the spirit of Zeta regularization, I will use complex powers of the elliptic operator giving rise to the Klein-Gordon operator to define a regularized Wightman field, whose pointwise evaluation is well-defined; this is used to define regularized observables (such as the stress-energy tensor), allowing to make contact with the theory of the Casimir effect. Renormalization is accomplished via analytic continuation of the resulting kernels. Finally, the computational efficiency of the above methods is exhibited with some explicit examples.

Joint work with Livio Pizzocchero.

References: arXiv:1505.00711, arXiv:1505.01044, arXiv:1505.01651, arXiv:1505.03276.

Bounds for complex eigenvalues of single and bilayer graphene's Hamiltonian operator

Francesco Ferrulli Imperial College London

Recently the increased quality in graphene's production as well as the thriving of applications of this new material, from nano-structures to macro compounds, has attracted the community of physicists, as well as mathematicians and engineers, to the study of its astonishing electronic properties. In the first section of the talk I will briefly recover the Dirac's like structure of the Hamiltonian operator for certain value of the energy close to the Fermi energy level for graphene via the tight binding approach. In the second part we will deal with complex potentials that will be seen as perturbations of the Hamiltonians derived in the first section. The non trivial complexity of them implies in a general context the loss of self-adjointness and the possibility for complex eigenvalues to appear. Inspired by the one-dimensional case [2] where it is possible to determine the region where the complex eigenvalues eventually lie, I will present some results obtained for the two-dimensional bilayer case, where even though is not possible to determine the exact region we succeed in determining some a-priori constraints for the complex eigenvalues.

- 1. A. A. Abramov, A. Aslanyan, and E. B. Davies, *Bounds on complex eigenvalues* and resonances, J. Phys. A 34 (1999), 57–72.
- 2. J.-C. Cuenin, A. Laptev, and C. Tretter, *Eigenvalue Estimates for Non-Selfadjoint Dirac Operators on the Real Line*, Annales Henri Poincare 15 (2014). 707–736.
- 3. K.S. Novoselov, and A.K. Geim, *The rise of Graphene*, Nature Materials 6 (2007), 183–191.

4. H. Raza, *Graphene Nanoelectronics: Metrology, Synthesis, Properties and Applications*, NanoScience and Technology, 2012, Springer Publsh.

Schrödinger equation with nonlinear defect as effective model

Domenico Finco Univesità Telematica Internazionale Uninettuno

We discuss the convergence of the evolution for a NLS with rescaled inhomogeneous nonlinearity to a NLS with concentrated nonlinearity in dimension one and dimensione three.

Gibbs Measures for the DNLS equation

Giuseppe Genovese Institut für Mathematik, Universität Zürich

The derivative non linear Schrödinger equation

 $i\partial_t \psi = \psi'' + i\beta(|\psi|^2\psi)', \quad \beta \in \mathbb{R},$

is an integrable infinite dimensional Hamiltonian system. In the space-periodic setting and for $|\beta|$ small enough, we present the construction of a sequence of functional measures associated to the integrals of motions, supported on Sobolev spaces of increasing regularity.

Joint work with R. Lucá (ICMAT, Madrid) and D. Valeri (YMSC, Beijing).

Approximation of Schrödinger operators with singular interactions supported on hypersurfaces

Markus Holzmann TU Graz

We discuss that Schrödinger operators with singular δ -interactions supported on hypersurfaces can be approximated by Hamiltonians with regular potentials. For this purpose, we construct a sequence of potentials V_{ε} such that the associated Schrödinger operators $H_{\varepsilon} := -\Delta - V_{\varepsilon}$ converge to a Hamiltonian with a singular interaction in the norm resolvent sense, as $\varepsilon \to 0+$; the connection of the potentials V_{ε} and the interaction strength of the limit operator can be stated explicitly. This implies, that the spectral properties of H_{ε} and those of the limit operator are approximately the same for sufficiently small ε . This yields a justification for the usage of Schrödinger operators with δ -interactions as idealized models to solve the spectral problem for classical Hamiltonians.

Essential spectrum of singular matrix differential operators

Orif Ibrogimov Mathematisches Institut, Universität Bern

We study the essential spectrum of symmetric systems of singular ordinary differential operators of mixed order. The essential spectrum is described explicitly and consists of two branches. One branch can be obtained as the limit of the essential spectra of the restrictions to compact subintervals exhausting to the original interval. The second branch is present for the original interval only and can be described by the singularities of the coefficients functions at the singular end-points. The efficacy of the results is demonstrated by applications to magnetohydrodynamics and astrophysics.

The results were obtained in collaboration with P. Siegl and C. Tretter (University of Bern, Switzerland).

Periodic operators with defects of smaller dimension

Anton A. Kutsenko Aarhus University

Multidimensional periodic operators with defects (MPOD) is a large class of operators which includes periodic Schrödinger operators with various defects, wave operators with guides, boundaries, local defects and so on. We show that MPOD form a Banach algebra of integral operators of a special form. We construct explicit traces and determinants in this algebra, and we consider some applications: an explicit procedure of finding spectrum, inverse problems, spectral estimates, numerical examples.

On asymptotic expansions for spin boson models

Markus Lange Friedrich-Schiller-University Jena

We will consider the Hamiltonian of a quantum mechanical system with finitely many degrees of freedom, which is linearly coupled to a field of relativistic massless bosons. This model is also known as generalized spin boson model. Assuming a natural infrared assumption, we will show that the ground state energy admits an asymptotic expansion to arbitrary order. This infrared assumption is weaker than the assumption usually needed for other methods such as operator theoretic renormalization to be applicable. The result complements previously shown analyticity properties.

Classification of topological insulators for classical light

Max Lein Advanced Institute of Materials Research, Tohoku University

Recently, topological insulators were experimentally realized with classical light in a number of spectacular experiments. In analogy to similar phenomena from solid state physics, unidirectional, backscattering-free edge currents were observed in periodic

light conductors such as photonic versions of graphene. My research aims at providing a first principles explanation of such dynamical and topological effects in *"photonic topological insulators"* (PTIs); Of particular interest are "photonic bulk-edge correspondences".

As a first step, we give an *exhaustive classification* of PTIs for the first time. In order to be able to adapt the Cartan-Altland-Zirnbauer classification scheme of topological insulators to electromagnetism it is necessary to recast the Maxwell equations as a "Schrödinger-type equation". It turns out that periodic light conductors made up of ordinary materials (such as silica) are of *class BDI*, meaning they resemble topological superconductors rather than analogous quantum systems from solid state physics. There is a second important difference to quantum systems: electromagnetic fields are *real*, and this gives rise to a *grading*. Consequently, *existing derivations* of, say, bulk-edge correspondences from solid state physics *do not automatically apply to PTIs*. Lastly, I will explain the ramifications this has for picking suitable effective tight-binding models.

In collaboration with Giuseppe De Nittis.

Spectral analysis of the magnetic Laplacian in the semiclassical limit

Jean-Philippe Miqueu University of Rennes 1

The spectral theory of the Schrödinger operator with magnetic field and semiclassical parameter generated a lot of interest. It is linked to the Ginzburg-Landau functional and applies to the physical study of surface superconductivity. The subject is now well known in two dimension, but most of the work concerns the case of non vanishing magnetic fields.

This talk will be devoted to the spectral analysis of a self-adjoint realization of the operator:

$$\mathcal{P}_{h,\mathbf{A},\Omega} = (-ih\nabla + \mathbf{A})^2 = \sum_{j=1}^2 (-ih\partial_{x_j} + A_j)^2,$$

in the semiclassical regime, where the magnetic field $\mathbf{B} = \partial_{x_1}A_2 - \partial_{x_2}A_1$ vanishes along a regular curve (with Ω a bounded and simply connected domain of \mathbb{R}^2 with smooth boundary and $\mathbf{A} \in \mathcal{C}^{\infty}(\overline{\Omega}, \mathbb{R}^2)$).

Such a cancellation brings the structure of the semiclassical limit. The aim is to explore the hierarchy of the model operators which would appear. We will be especially interested in the first asymptotic term of the lowest eigenvalue $\lambda_1(h)$, when the parameter *h* goes to 0. The talk will be illustrated by numerical simulations based on the library "Mélina++" (finite elements) developed at the University of Rennes 1.

Non-equilibrium physics in a quenched Luttinger model

Per Moosavi KTH Royal Institute of Technology, Stockholm

Exact analytical results are presented for a system of interacting spinless fermions in 1+1D described by the Luttinger model with a short range non-local interaction. The system is studied out of equilibrium, and we show that its evolution is ballistic but dispersive and that it approaches a final steady state following a quench from a domain wall initial state. This final steady state carries a current and has an effective chemical potential difference obtained from the two-point correlation function. Both depend on the microscopic details of the model but we show that the corresponding conductance is universal.

This work is in collaboration with E. Langmann, J. L. Lebowitz, and V. Mastropietro (arXiv:1511.01884).

Effective dynamics for a multi-component mixture of Bose-Einstein condensates

Alessandro Olgiati SISSA Trieste

I will present a rigorous derivation of the effective evolution equations for the manybody Schrödinger dynamics of a system consisting of different species of identical bosons (a so-called "mixture" of interacting BEC). In the limit of large system, the effective dynamics is ruled by coupled nonlinear Schrödinger equations, one for each single-particle orbital of the BEC. Moreover, a quantitative error will be provided, in terms of the actual number of particles for each species, when the large system is replaced with an ideally infinite one. This result is obtained by a suitable adaptaion of Pickl's counting particles method within the reduced density matrix formalism.

On the bound states of Schrödinger operators with δ -interactions on conical surfaces

Thomas Ourmières-Bonafos BCAM-Basque Center for Applied Mathematics

Hamiltonians in strong homogeneous magnetic fields or photonics crystals with high-contrast are approximated by Schrödinger operators with δ -type interactions supported on sets of zero Lebesgue measure (points, curves, surfaces or hypersurfaces). In quantum mechanics, the spectrum of such Schrödinger operators is related to admissible values of the energy and a natural issue is to understand how the geometry of the support of the δ -interaction influences the spectrum.

In this talk we consider, in dimension greater than or equal to three, a Laplacian coupled with an attractive δ -interaction supported on a cone whose cross section is the sphere of co-dimension two.

We prove that there is discrete spectrum only in dimension three and that, in this case, the eigenvalues are non-decreasing functions of the aperture of the cone. The

main result of this work is the exhibition of the precise logarithmic accumulation of the discrete spectrum below the threshold of the essential spectrum.

Joint work with Vladimir Lotoreichik.

1. V. Lotoreichik, T. Ourmières-Bonafos. On the bound states of Schrödinger operators with δ -interactions on conical surfaces. preprint ArXiv (2015)

Geometric description of finite-dimensional quantum systems in complex projective spaces and applications

Davide Pastorello University of Trento and INFN

Adopting a geometric classical-like point of view on Quantum Mechanics is an intriguing idea since we know that geometric methods are very powerful in Classical Mechanics thus we can try to use them to study quantum systems. This talk is focused on geometric Hamiltonian formulation of finite-dimensional quantum mechanics where phase space is given by the projective space (as a Kähler manifold) constructed out from Hilbert space of the considered quantum theory [2]. Within such a framework quantum observables are represented by phase space functions, quantum states are described by Liouville densities (phase space probability densities), and Schrödinger dynamics is induced by the flow of a Hamiltonian vector field w.r.t. a natural symplectic structure.

Some applications of geometric Hamiltonian picture will be presented like definition of an entanglement measure [1] and a new approach to quantum control theory.

- 1. D. Pastorello. A geometric Hamiltonian description of composite quantum systems and quantum entanglement, International Journal of Geometric Methods in Modern Physics Vol. 12, No. 07, 1550069 (2015)
- V. Moretti and D. Pastorello. Frame functions in finite-dimensional Quantum Mechanics and its Hamiltonian formulation on complex projective spaces, International Journal of Geometric Methods in Modern Physics (in print) DOI: 10.1142/S0219887816500134

Hardy inequalities on the Heisenberg group for convex bounded polytopes

Bartosch Ruszkowski Institut für Analysis, Dynamik und Modellierung, Universität Stuttgart

In the Euclidean setting it is a matter of folklore that for a given bounded convex domain $\Omega \subset \mathbb{R}^n$ Hardy's inequality holds with an optimal constant independent of Ω , where the weight function is the Euclidean distance to the boundary of Ω . The question

arises whether such inequalities still hold if we consider more difficult operators than the Laplacian.

In this talk we discuss this problem in the setting of the first Heisenberg group \mathbb{H} . We prove a Hardy-type inequality for the Heisenberg Laplacian on open bounded convex polytopes. The integral weight of the Hardy inequality is given by the distance function to the boundary measured with respect to the Carnot-Carathéodory metric. The constant depends on the number of hyperplanes, which are not orthogonal to the hyperplane $x_3 = 0$.

Joint work with Hynek Kovařík and Timo Weidl.

From the Hartree-Fock dynamics to the Vlasov equation

Chiara Saffirio Institut für Mathematik, Universität Zürich

We will discuss the convergence (in the semiclassical limit) of a solution to the Hartree-Fock equation towards an operator, whose Wigner transform is a solution to the Vlasov equation. We will consider both cases of positive and zero temperature.

This is a joint work with N. Benedikter, M. Porta and B. Schlein.

A sub-Riemannian Santaló formula and applications

Marcello Seri

We prove a sub-Riemannian version of the classical Santaló formula: a result in integral geometry that describes the intrinsic Liouville measure on the unit cotangent bundle in terms of the geodesic flow. Due to time restriction we will focus on the meaning of the result and describe some direct applications like (p-)Hardy -type and isoperimetric-type inequalities for a compact domain with sufficiently regular boundary, and curvature independent lower bound for the first Dirichlet eigenvalue of the intrinsic sub-Laplacian and of magnetic Laplace-Beltrami operators. If time permits we will discuss how this leads to prove quantum trapping by volume explosion.

Joint works with D. Prandi and L. Rizzi.

Topological edge states in two-gap unitary systems: a transfer matrix approach

Clément Tauber ENS de Lyon

Several physical systems such as photonic networks or periodically driven systems are described by a unitary family of operators rather than a Hermitian one. For systems with translation invariance, Bloch bands and gaps also arise in that context and the computation of the corresponding topological invariants reveals subtleties that are absent in Hermitian systems. I will present a simple unitary model where all the bulk invariants (Chern numbers) are trivial, but still presenting robust edge modes on a cylinder geometry. Using the transfer matrix formalism, it is possible to construct a topological invariant associated to such edge modes as a winding number around a Riemann surface naturally constructed from analyticity considerations on the solutions of the problem.

NLS on graphs: recent results and perspectives

Lorenzo Tentarelli Politecnico di Torino

I will make a brief overview about recent works on stationary solutions of the *Gross– Pitaevskii* equation (in the *focusing* case) on *noncompact metric graphs*. In particular, I will focus on some results (obtained in collaboration with E. Serra) concerning the case where the nonlinearity is localized only on a *compact portion* of the graph. Precisely, I will discuss existence and nonexistence of *ground states*, defined as constrained minimizers of the associated NLS energy, and then (existence and) multiplicity of *bound states*, arising as constrained critical points of the energy functional at higher levels. Finally, I will point out some possible guidelines for future research on this topics.

Entanglement Rates in Bipartite Systems

Anna Vershynina Technical University of Munich, Germany

Entanglement rate in a bipartite system is the maximal rate at which an entanglement can be generated in time. The goal is to find an upper bound to this rate. First, I will review the problem in closed ancilla-assisted systems give an overview of its solution. Then I will move to the open systems, where the generator of irreversible dynamics consists of a Hamiltonian and dissipative terms in Lindblad form. The relative entropy of entanglement is chosen as a measure of entanglement in an ancilla-free system. I will prove an upper bound on the entangling rate that has a logarithmic dependence on a dimension of a smaller system in a bipartite cut. I will also investigate the rate of change of quantum mutual information in an ancilla-assisted system and provide an upper bound independent of dimension of ancillas.

On the semiclassical limiting eigenvalue distribution for the hydrogen atom in a constant direction magnetic field

Carlos Villegas-Blas

Universidad Nacional Autonoma de Mexico, Instituto de Matematicas

We consider the Hamiltonian of the Zeeman hydrogen atom H_Z given by the hydrogen atom Hamiltonian (with the Planck's parameter \hbar included) under the influence of a constant magnetic field. Considering \hbar taking values on the discrete set h = 1/(N+1), $N = 0, 1, 2, \ldots$ and the strength of the magnetic field depending on \hbar , we show, for N sufficiently large, the existence of clusters of eigenvalues of H_Z around the real number E = -1/2. The clusters are parametrized by N and the total multiplicity of eigenvalues within a cluster grows like $(N + 1)^2$.

Then we study the distribution of eigenvalues inside the clustres with $N \mapsto \infty$ obtaining a weak limit measure involving the component of the classical angular momentum vector in the direction of the magnetic field. This measure is related with the Liouville measure on the surface of constant energy E = -1/2 for the Kepler problem. The use of suitable coherent states will be described.

We will also obtain a weak limit measure when the eigenvalue of the component of the angular momentum operator along the magnetic field is constant and then we actually have existence of sub-clusters. A reduction process will be involved.

This is joint work with Misael Avendao-Camacho and Peter Hislop.

Schrödinger dynamics and optimal transport of measures on the flat torus

Lorenzo Zanelli University of Padova

The aim of this talk is to show a link between the semiclassical Analysis of Schrödinger equations and the Optimal Transport theory. We show that the projection on \mathbb{T}^n of some time depent semiclassical measures on $\mathbb{T}^n \times \mathbb{R}^n$ are optimal displacement interpolations. Moreover, we show the completing viewpoint by proving that a family of optimal displacement interpolations can be viewed as projected time dependent semiclassical measures.

List of Participants

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