On the time dependent Ginzburg-Landau system.

Bernard Helffer
LMJL Université de Nantes,
FRANCE

In this course we would like to discuss spectral properties of some non self-adjoint operators appearing in the analysis of the long time behavior of the solutions of the time dependent Ginzburg Landau system (due to Eliashberg-Gorkov) and then to consider the question of the global stability of the stationary normal solutions in presence of an electric current flowing through a two-dimensional wire.
We show that when the current is sufficiently strong the solution converges in the long-time limit to the normal state. We provide two types of upper bounds for the critical current where such global stability is achieved: by using the principal eigenvalue of the magnetic Laplacian associated with the normal magnetic field, and through the norm of the resolvent of the linearized steady-state operator. In the latter case we estimate the resolvent norm in large domains by the norms of approximate operators defined on the plane and the half-plane. We also obtain a lower bound, in large domains, for the above critical current by obtaining the current for which the normal state looses its local stability.

The recent results presented in the course were obtained in collaboration with Y. Almog or X. Pan. Introductory books for the time independent problems could be the monographs of Sandier-Serfaty and Fournais-Helffer (both in the series Progress in non linear analysis – Birkhäuser).

References

[1] Y. Almog, *The stability of the normal state of superconductors in the presence of electric currents*. SIAM Journal on Mathematical Analysis, 40 (2008), pp. 824–850.

[2] Y. Almog and B. Helffer. *Global stability of the normal state of superconductors in the presence of a strong electric current.* Comm. in Math. Physics 2014.

[3] Y. Almog and B. Helffer On the spectrum of non-selfadjoint Schrödinger operators with compact resolvent. arXiv:1410.5122. Comm. in PDE 2015.

[4] Y. Almog, B. Helffer and X. B. Pan, *Superconductivity near the normal state under the action of electric currents and induced magnetic fields in* $\mathbb{R}^2$, Commun. Math. Phys. 300 (2010), 147-184.

[5] Y. Almog, B. Helffer and X. B. Pan, *Superconductivity near the normal state in a half-plane under the action of a perpendicular electric current and an induced fagnetic field*, Trans. AMS 365 (2013), 1183-1217.

[6] Y. Almog, B. Helffer and X. B. Pan, *Superconductivity near the normal state in a half-plane under the action of a perpendicular current and an induced magnetic field II : the large conductivity limit,* Siam J. Math. Anal. 44, No. 6 (2012), 3671-3733.

[7] Y. Almog, B. Helffer and X. B. Pan, Mixed normal-superconducting states in the presence of strong electric currents. ArXiv:1505.06327 (2015).

[8] P. Bauman, H. Jadallah and D. Phillips. *Classical solutions to the time-dependent Ginzburg-Landau equations for a bounded superconducting body in a vacuum*, J. Math. Phys., **46** (2005), 095104.

[9] Z. M. Chen, K.-H. Hoffmann, and J. Liang. *On a nonstationary Ginzburg-Landau superconductivity model*, Math. Methods Appl. Sci., 16 (1993), pp. 855–875.

[10] R. Dautray and J.L. Lions. *Mathematical Analysis and Numerical Methods for Science and Technology.* Vol. 5, Evolution problems I, Springer-Verlag, 1988.

[11] Q. Du. *Global existence and uniqueness of solutions of the time-dependent Ginzburg-Landau model for superconductivity*, Appl. Anal., 53 (1994), pp. 1–17.

[12] Q. Du and P. Gray. *High-kappa limits of the time-dependent Ginzburg-Landau model*, SIAM J. Appl. Math., 56 (1996), pp. 1060–1093.

[13] K.J. Engel and R. Nagel. *One-parameter semigroups for linear evolution equations.* Graduate texts in Mathematics 194, Springer.

[14] L. C. Evans. *Partial Differential Equations*, AMS, 1st. ed., 1998.

[15] E. Feireisl and P. Takáč. *Long-time stabilization of solutions to the Ginzburg-Landau equations of superconductivity*, Monatsh. Math., 133 (2001), pp. 197–221.

[16] J. Fleckinger-Pellé, H. G. Kaper, and P. Takáč. *Dynamics of the Ginzburg-Landau equations of superconductivity*, Nonlinear Anal., 32 (1998), pp. 647–665.

[17] S. Fournais and B. Helffer. *Spectral Methods in Surface Superconductivity*, Birkhäuser, 2009.

[18] T. Giorgi and D. Philips. *The breakdown of superconductivity due to strong fields for the Ginzburg-Landau model*, SIAM J. Math. Anal., 30 (1999), pp. 341–359.

[19] P. Grisvard. *Elliptic problems in nonsmooth domains*, vol. 24 of Monographs and Studies in Mathematics, Pitman (Advanced Publishing Program), Boston, MA, 1985.

[20] P. Grisvard. *Singularities in boundary value problems.* Springer (1992).

[21] B. Helffer, On pseudo-spectral problems related to a time dependent model in superconductivity with electric current. Confluentes Math. 3, No. 2, 237-251 (2011).

[22] B. Helffer. Spectral theory and its applications. Cambridge University Press 2013.

[23] B. Helffer and J. Sjöstrand. *From resolvent bounds to semigroup bounds.* Preprint : arXiv:1001.4171v1, (2010).

[24] R. Henry. Master's memoir (2010).

[25] R. Henry. *Spectral instability of some non-selfadjoint anharmonic oscillators.* C. R. Acad. Sci. Paris, Ser. I 350 (2012) 1043-1046.

[26] R. Henry. *Spectral instability for the complex Airy operator and even non-selfadjoint anharmonic oscillators.* To appear in Journal of Spectral Theory.

[27] R. Henry. *On the semiclassical analysis of Schrödinger operators with purely imaginary electric potentials in a bounded domain.* To appear in CPDE (2015).

[28] B. I. Ivlev and N. B. Kopnin. *Electric currents and resistive states in thin superconductors*, Advances in Physics, 33 (1984), pp. 47–114.

[29] R. Montgomery. *Hearing the zero locus of a magnetic field*, Comm. Math. Phys., 168 (1995), pp. 651–675.

[30] X.-B. Pan and K.-H. Kwek. *Schrödinger operators with non-degenerately vanishing magnetic fields in bounded domains*, Trans. Amer. Math. Soc., 354 (2002), pp. 4201–4227 (electronic).

[31] J. Rubinstein, P. Sternberg, and J. Kim. *On the behavior of a superconducting wire subjected to a constant voltage difference*, SIAM Journal on Applied Mathematics, 70 (2010), pp. 1739–1760.

[32] J. Rubinstein, P. Sternberg, and K. Zumbrun. *The Resistive State in a Superconducting Wire: Bifurcation from the Normal State*, Archive for Rational Mechanices and Analysis, 195 (2010), pp. 117–158.

[33] I. Tice. *Ginzburg-Landau vortex dynamics driven by an applied boundary current*, Comm. Pure Appl. Math., 63 (2010), pp. 1622–1676.