Non-equilibrium physics in a quenched Luttinger model

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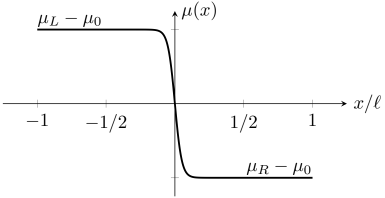
Bressanone, February 8, 2016

[1] arXiv:1511.01884 (2015)

Mesoscopic 1+1D system

Evolution and approach to steady state of a 1+1D system of interacting spinless fermions from a domain wall initial state...



Chemical potential   Temperature

considering a subsystem $[-\ell, \ell]$ with lengths $L > \ell > 0$.

For XX and XXZ models

Antal et al., PRE 59 (1999); Rigol et al., PRL 98 (2007)
Lancaster, Mitra, PRE 81 (2010); Sabetta, Misguich, PRB 88 (2013); Liu, Andrei, PRL 112 (2014)

Electric transport in the Luttinger model

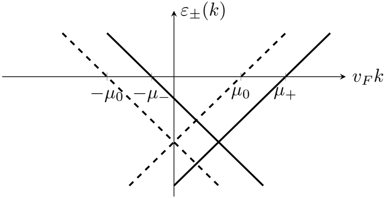
Steady current in the final state

$$I = G_{\lambda,\lambda'}(\mu_L - \mu_R) = G(\mu_+ - \mu_-)$$

μ_L left and μ_R right chemical potentials

μ_+ right- and μ_- left-moving fermions

Fermi sea in the final state



(example for $\lambda = \lambda' = 0$)

$G_{\lambda,\lambda} = K_\lambda e^2/h$ renormalized

Kane, Fisher, PRL 68 (1992)

$G = e^2/h$ universal

Maslov, Stone, PRB 52 (1995)

Kawabata, J. Phys. Soc. Jpn. 65 (1996); Alekseev, Cheianov, Fröhlich, PRB 54 (1996), PRL 81 (1998)

Outline

- Introduction
- Equilibrium Luttinger model
- Non-equilibrium Luttinger model
- Approach to steady state
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Luttinger model

Luttinger Hamiltonian

$$H_\lambda = \sum_{r=\pm} \int_{-L/2}^{L/2} dx \, {:}\psi_r^\dagger(x) \, (-irv_F\partial_x) \, \psi_r(x){:}$$
$$+ \lambda \sum_{r,r'=\pm} \int_{-L/2}^{L/2} \mathrm{d}x \, \mathrm{d}y \, {:}\psi_r^\dagger(x)\psi_r(x){:} V(x-y) {:}\psi_{r'}^\dagger(y)\psi_{r'}(y){:}$$

with short range, non-local potential $V(x)$ and coupling λ satisfying (*).

Tomonaga, Prog. Theor. Phys. 5 (1950); Luttinger, JMP 4 (1963); Mattis, Lieb, JMP 6 (1965)

$|\Psi_\lambda\rangle$ = Ground state; ${:}(\cdots){:}$ = Normal ordering

(*) In Fourier space $p \in (2\pi/L)\mathbb{Z}$:

$$\hat{V}(p) = \hat{V}(-p), \qquad \lambda\hat{V}(p) > -\pi v_F/2, \qquad \hat{V}(p)|p|^{1+\varepsilon} \xrightarrow{|p|\to\infty} 0, \ \varepsilon > 0.$$

Equilibrium correlation functions

Renormalized Fermi velocity

$$v_\lambda(p) = v_F \sqrt{1 + 2\lambda \hat{V}(p)/\pi v_F}$$

Luttinger parameter

$$K_\lambda(p) = 1/\sqrt{1 + 2\lambda \hat{V}(p)/\pi v_F}$$

Equilibrium two-point correlation function in the thermodynamic limit

$$\langle \Psi_\lambda | \psi_r^\dagger(x) \psi_r(y) | \Psi_\lambda \rangle = \frac{i}{2\pi r(x-y) + i0^+} \exp\left(\int_0^\infty \mathrm{d}p \, \frac{\eta_\lambda(p)}{p} (\cos p(x-y) - 1) \right)$$

Equilibrium exponent

$$\eta_\lambda(p) = \frac{K_\lambda(p) + K_\lambda(p)^{-1}}{2} - 1$$

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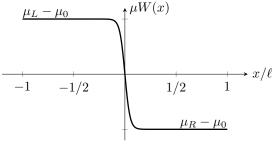
Quenched Luttinger model

Quantum quench $H_\lambda \to H_{\lambda'}$ from a uniform initial state for special cases $\lambda = 0 \neq \lambda'$ and $\lambda \neq 0 = \lambda'$.

Cazalilla, PRL 97 (2006); Iucci, Cazalilla, PRA 80 (2009); Mastropietro, Wang, PRB 91 (2015)

Luttinger Hamiltonian producing an initial domain wall density profile

$$H_{\lambda,\mu} = H_\lambda - \mu \sum_{r=\pm} \int_{-L/2}^{L/2} \mathrm{d}x\, W(x) :\psi_r^\dagger(x)\psi_r(x):$$



using an external field $W(x)$.

Quantum quench $H_{\lambda,\mu} \to H_{\lambda',0}$ from a domain wall initial state for all possible λ and λ'.

Langmann, Lebowitz, Mastropietro, PM, arXiv:1511.01884 (2015)

Local observables

Evolution $|\Psi^{\lambda'}_{\lambda,\mu}(t)\rangle = e^{-iH_{\lambda'}t}|\Psi_{\lambda,\mu}\rangle$ of the ground state $|\Psi_{\lambda,\mu}\rangle$ of $H_{\lambda,\mu}$.

- ◇ Compute expectation values of observables for finite L.
- ◇ Take thermodynamic limit $L \to \infty$.
- ◇ Study asymptotic behavior of observables as $t \to \infty$.

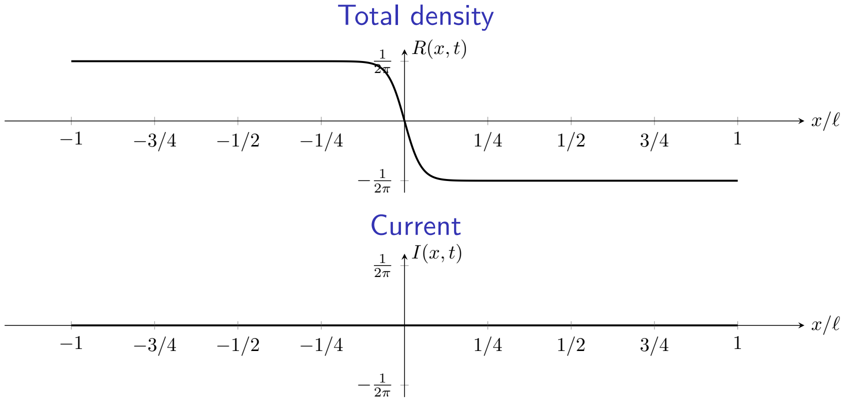
Total density and current

$$R(x,t) = \langle \Psi^{\lambda'}_{\lambda,\mu}(t)|\rho(x)|\Psi^{\lambda'}_{\lambda,\mu}(t)\rangle, \qquad \rho(x) = \rho_+(x) + \rho_-(x)$$

$$I(x,t) = \langle \Psi^{\lambda'}_{\lambda,\mu}(t)|j(x)|\Psi^{\lambda'}_{\lambda,\mu}(t)\rangle, \qquad j(x) = v_F(\rho_+(x) - \rho_-(x))$$

Densities $\rho_r(x) = {:}\psi^\dagger_r(x)\psi_r(x){:}$

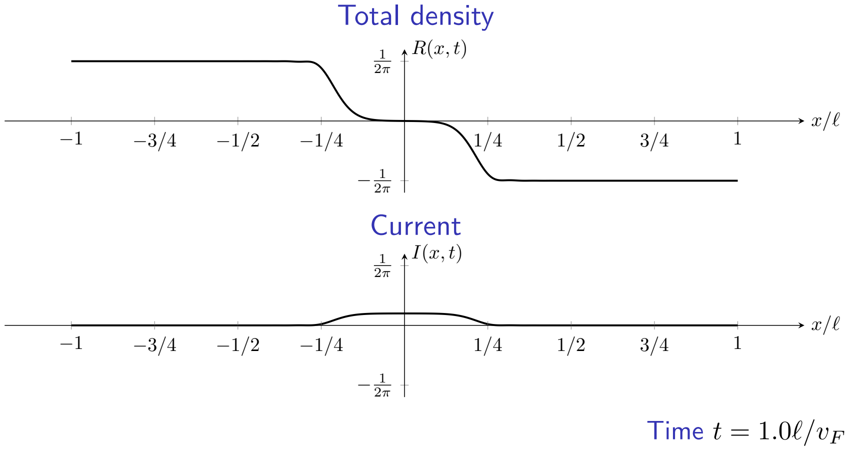
Evolution following a quench



Time $t = 0.0\ell/v_F$

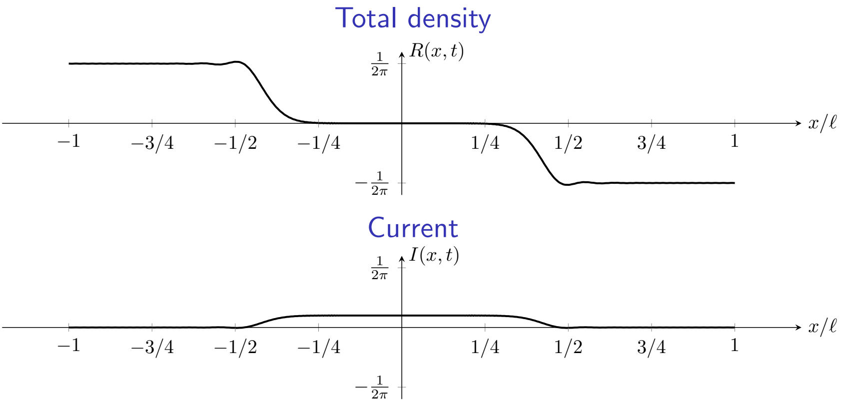
Ground state of $H_{\lambda,\mu}$ evolved under $H_{\lambda'}$ for $\lambda = 0$, $\lambda' = -0.96$, and $\mu = 1$ with interaction potential $\hat{V}(p) = (\pi v_F/2)\,\mathrm{sech}(ap)$ where $a = 0.0025\ell$ and $v_F = 1$.

Evolution following a quench



Ground state of $H_{\lambda,\mu}$ evolved under $H_{\lambda'}$ for $\lambda = 0$, $\lambda' = -0.96$, and $\mu = 1$ with interaction potential $\hat{V}(p) = (\pi v_F/2)\,\mathrm{sech}(ap)$ where $a = 0.0025\ell$ and $v_F = 1$.

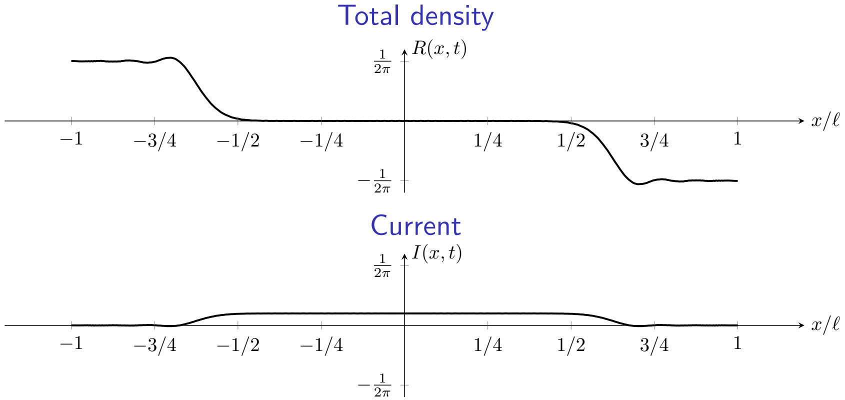
Evolution following a quench



Time $t = 2.0\ell/v_F$

Ground state of $H_{\lambda,\mu}$ evolved under $H_{\lambda'}$ for $\lambda = 0$, $\lambda' = -0.96$, and $\mu = 1$ with interaction potential $\hat{V}(p) = (\pi v_F/2)\,\mathrm{sech}(ap)$ where $a = 0.0025\ell$ and $v_F = 1$.

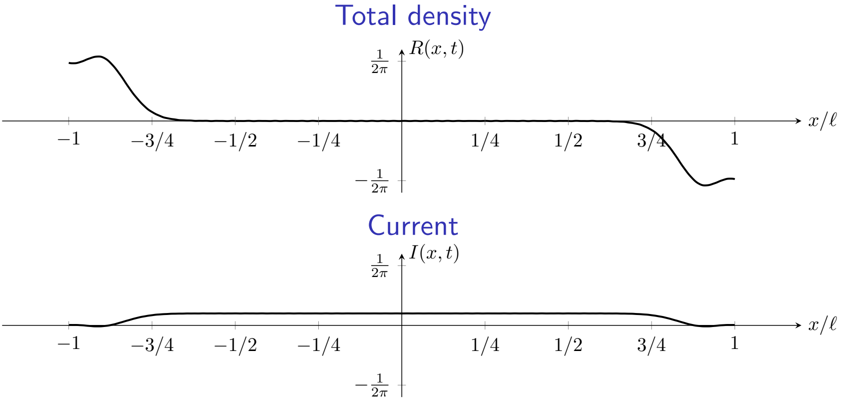
Evolution following a quench



Time $t = 3.0\ell/v_F$

Ground state of $H_{\lambda,\mu}$ evolved under $H_{\lambda'}$ for $\lambda = 0$, $\lambda' = -0.96$, and $\mu = 1$ with interaction potential $\hat{V}(p) = (\pi v_F/2)\,\mathrm{sech}(ap)$ where $a = 0.0025\ell$ and $v_F = 1$.

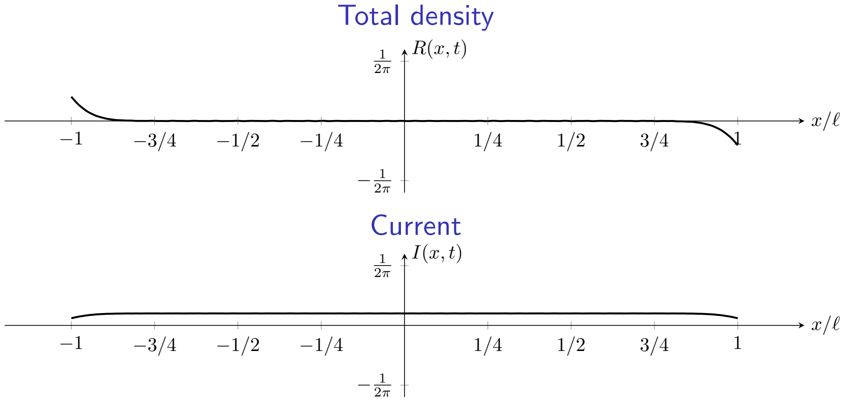
Evolution following a quench



Time $t = 4.0\ell/v_F$

Ground state of $H_{\lambda,\mu}$ evolved under $H_{\lambda'}$ for $\lambda = 0$, $\lambda' = -0.96$, and $\mu = 1$ with interaction potential $\hat{V}(p) = (\pi v_F/2)\,\mathrm{sech}(ap)$ where $a = 0.0025\ell$ and $v_F = 1$.

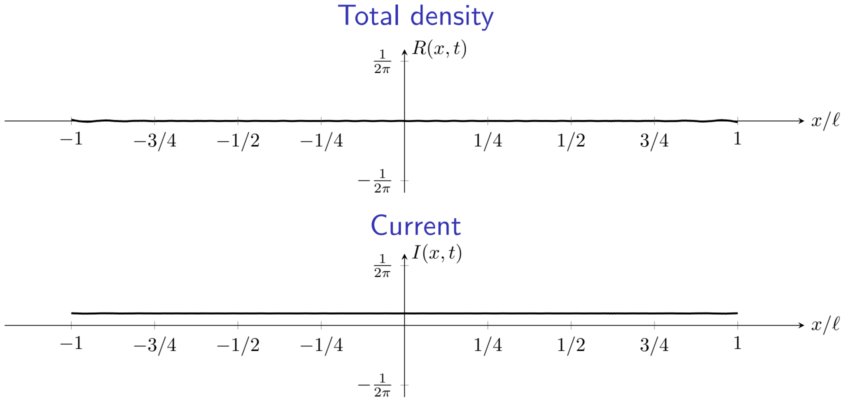
Evolution following a quench



Time $t = 5.0\ell/v_F$

Ground state of $H_{\lambda,\mu}$ evolved under $H_{\lambda'}$ for $\lambda = 0$, $\lambda' = -0.96$, and $\mu = 1$ with interaction potential $\hat{V}(p) = (\pi v_F/2)\,\mathrm{sech}(ap)$ where $a = 0.0025\ell$ and $v_F = 1$.

Evolution following a quench



Time $t = 6.0\ell/v_F$

Ground state of $H_{\lambda,\mu}$ evolved under $H_{\lambda'}$ for $\lambda = 0$, $\lambda' = -0.96$, and $\mu = 1$ with interaction potential $\hat{V}(p) = (\pi v_F/2)\,\mathrm{sech}(ap)$ where $a = 0.0025\ell$ and $v_F = 1$.

Non-equilibrium correlation functions

Two-point correlation function in the thermodynamic limit

$$\langle \Psi^{\lambda'}_{\lambda,\mu}(t) | \psi^\dagger_r(x) \psi_r(y) | \Psi^{\lambda'}_{\lambda,\mu}(t) \rangle = e^{-irv_F^{-1} A_r(x,y,t)(x-y)} S_r(x,y,t)$$

$A_r(x,y,t) =$ contribution from external field

$$\begin{aligned} S_r(x,y,t) &= \langle \Psi^{\lambda'}_{\lambda,0}(t) | \psi^\dagger_r(x) \psi_r(y) | \Psi^{\lambda'}_{\lambda,0}(t) \rangle \\ &= \frac{i}{2\pi r(x-y) + i0^+} \exp\left(\int_0^\infty \mathrm{d}p \, \frac{\eta_{\lambda,\lambda'}(p) - \gamma_{\lambda,\lambda'}(p) \cos(2pv_{\lambda'}(p)t)}{p} (\cos p(x-y) - 1) \right) \end{aligned}$$

Non-equilibrium exponents

$$\eta_{\lambda,\lambda'}(p) = \frac{K_\lambda(p)(K_{\lambda'}(p)^{-2} + 1) + K_\lambda(p)^{-1}(K_{\lambda'}(p)^2 + 1)}{4} - 1$$

$$\gamma_{\lambda,\lambda'}(p) = \frac{K_\lambda(p)(K_{\lambda'}(p)^{-2} - 1) + K_\lambda(p)^{-1}(K_{\lambda'}(p)^2 - 1)}{4}$$

Different from the equilibrium exponents if $\lambda \neq \lambda' \neq 0$.

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Asymptotic behavior

Theorem

If the interaction satisfies (*), then, in the thermodynamic limit,

$$\lim_{t\to\infty} R(x,t) = 0, \qquad \lim_{t\to\infty} I(x,t) = \frac{\mu}{2\pi}\frac{K_\lambda v_{\lambda'}}{v_\lambda}, \qquad \lim_{t\to\infty} A_r(x,y,t) = r\frac{\mu}{2}\frac{K_\lambda v_{\lambda'}}{v_\lambda}$$

and

$$\lim_{t\to\infty} S_r(x,y,t) = \frac{i}{2\pi r(x-y)+i0^+}\exp\left(\int_0^\infty \mathrm{d}p\,\frac{\eta_{\lambda,\lambda'}(p)}{p}\left(\cos p(x-y)-1\right)\right)$$

with $K_\lambda = K_\lambda(p=0)$ *and* $v_\lambda = v_\lambda(p=0)$.

Langmann, Lebowitz, Mastropietro, PM, arXiv:1511.01884 (2015)

(*) In Fourier space $p \in (2\pi/L)\mathbb{Z}$:

$$\hat{V}(p) \in C^2(\mathbb{R}), \quad \frac{\mathrm{d}^n\hat{V}(p)}{\mathrm{d}p^n} \in L^1(\mathbb{R}),\ n=0,1,2, \quad \lambda p\frac{\mathrm{d}\hat{V}(p)}{\mathrm{d}p} > -\pi v_F - 2\lambda\hat{V}(p).$$

Generalized canonical ensemble

Our system reaches a final steady state $\implies$ there is equilibration, but it is not the ground state of $H_{\lambda'}$ $\implies$ there is not thermalization.

To describe the final state in integrable systems

- usual canonical ensemble generally not sufficient,
- need generalized canonical ensemble with more conserved quantities.

Rigol et al., PRL 98 (2007); Eisert, Friesdorf, Gogolin, Nat. Phys. 11 (2015)

$\diamond$ $\lambda = \lambda'$: Generalized canonical ensemble with conserved quant. H_λ and Q_r given by Gibbs measure $e^{-\beta H}$ as $\beta \to \infty$ where

$$H = H_\lambda - \sum_{r=\pm} (\mu_r - \mu_0) Q_r, \qquad Q_r = \int_{-L/2}^{L/2} \mathrm{d}x \, \rho_r(x), \qquad \sum_{r=\pm} \mu_r = 2\mu_0.$$

$\diamond$ $\lambda \neq \lambda' \neq 0$: Infinite number of conserved quant. needed. Left open.

Iucci, Cazalilla, PRA 80 (2009); Langmann, Lebowitz, Mastropietro, PM, arXiv:1511.01884 (2015)

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Electrical conductance

Steady current: asymptotically in time

$$I = \lim_{t \to \infty} I(x,t) = \frac{\mu}{2\pi} \frac{K_\lambda v_{\lambda'}}{v_\lambda}$$

Chemical potentials: two-point correlation functions suggest to identify

$$\mu_r - \mu_0 = \lim_{t \to \infty} A_r(x,y,t) = r \frac{\mu}{2} \frac{K_\lambda v_{\lambda'}}{v_\lambda}$$

Renormalizations of I and $\mu_+ - \mu_-$ cancel $\implies$ electrical conductance

$$G = \frac{I}{\mu_+ - \mu_-} = \frac{(\mu/2\pi) K_\lambda v_{\lambda'}/v_\lambda}{\mu K_\lambda v_{\lambda'}/v_\lambda} = \frac{1}{2\pi} = \frac{e^2}{h}$$

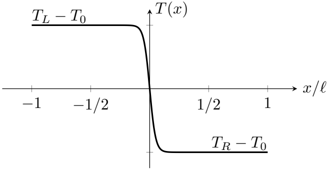
is universal. (units where $e = \hbar = 1$)

Thermal conductance

CFT approach $\Longrightarrow$ thermal conductance

$$K = \pi T_0/6$$

is universal for no or local interaction.



Bernard, Doyon, J. Phys. A: Math. Theor. 45 (2012), AHP 16 (2015)

Rieder, Lebowitz, Lieb, JMP 8 (1967); Spohn, Lebowitz, CMP 54 (1977); Kane, Fisher, PRL 76 (1996)
Aschbacher, Pillet, J. Stat. Phys. 112 (2003); De Luca, Viti, Bernard, Doyon, PRB 88 (2013)

Lorenz number is then also universal

$$L_{\mathsf{WF}} = \frac{K}{T_0 G} = \frac{\pi T_0/6}{T_0/2\pi} = \frac{\pi^2}{3}$$

as in Wiedemann-Franz law for 1+1D system without spin.

Question: What happens for non-local interaction?

Summary

- ◇ Exact analytical results for the evolution of the Luttinger model with short range, non-local interaction following a quench.
- ◇ Approach to final steady state
  - electrical conductance $G = I/(\mu_+ - \mu_-) = e^2/h$ is universal
  - non-equilib. exponents in general different from equilib. exponents
- ◇ Generalized canonical ensemble
  - $\lambda = \lambda'$: two-point correlation functions agree
  - $\lambda \neq \lambda' \neq 0$: left open
- ◇ Thermal conductance $K = ?$

Thank you for your attention!